

CONCURSUL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”

ETAPA LOCALĂ

8 februarie 2020

BAREM

CLASA A IX-A

(3 ore/săptămână)

| | | |
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| 1.) | Din oficiu | 1p |
| | a) $(n-1) + n + (n+1) = 2019$ | 2p |
| | $3n = 2019 \Rightarrow n = 673 \Rightarrow 2019 = 672 + 673 + 674$ | 2p |
| | b) $(n-3) + (n-2) + (n-1) + n + (n+1) + (n+2) + (n+3) = \overline{c_1 c_2 \dots c_k 2020}$ | 2p |
| | $9n = \overline{c_1 c_2 \dots c_k 2020} \Rightarrow (c_1 + c_2 + \dots + c_k + 4) : 9$ \Rightarrow cel mai mic număr este 52020 | 3p |

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| 2.) | Din oficiu | 1p |
| | $\frac{x+2 \cdot 2020}{2021} + \frac{x+2 \cdot 2021}{2022} + \frac{x+2 \cdot 2022}{2023} + \dots + \frac{x+2 \cdot (2019+c)}{2020+c} = \frac{2+2+\dots+2}{c \text{ termeni}}$ | 2p |
| | $\left(\frac{x+2 \cdot 2020}{2021} - 2 \right) + \left(\frac{x+2 \cdot 2021}{2022} - 2 \right) + \dots + \left(\frac{x+2 \cdot (2019+c)}{2020+c} - 2 \right) = 0$ | 3p |
| | $\left(\frac{x-2}{2021} \right) + \left(\frac{x-2}{2022} \right) + \dots + \left(\frac{x-2}{2020+c} \right) = 0$ | 2p |
| | $\Rightarrow (x-2) \underbrace{\left(\frac{1}{2021} + \frac{1}{2022} + \dots + \frac{1}{2020+c} \right)}_{\neq 0} = 0 \Rightarrow x = 2$ | 2p |

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| 3.) | Din oficiu | 1p |
| | a) $\sqrt{xy} \geq \frac{2xy}{x+y} \Rightarrow 1 \geq \frac{2\sqrt{xy}}{x+y} \Rightarrow x+y \geq 2\sqrt{xy}$ | 2p |
| | $x+y-2\sqrt{xy} \geq 0 \Rightarrow (\sqrt{x}-\sqrt{y})^2 \geq 0$ | 2p |
| | b) $\sqrt{\frac{a}{b+c}} \cdot 1 \geq \frac{2 \cdot \frac{a}{b+c} \cdot 1}{\frac{a}{b+c} + 1} \Rightarrow \sqrt{\frac{a}{b+c}} \geq \frac{2a}{a+b+c}$ | 3p |
| | Analog: $\sqrt{\frac{b}{a+c}} \geq \frac{2b}{a+b+c}$, $\sqrt{\frac{c}{a+b}} \geq \frac{2c}{a+b+c}$ | 1p |
| | $\sqrt{\frac{a}{b+c}} + \sqrt{\frac{b}{a+c}} + \sqrt{\frac{c}{a+b}} \geq \frac{2a+2b+2c}{a+b+c} \geq 2$ | 1p |

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| 4.) | Din oficiu | 1p |
| |  | 2p |

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| | $\overrightarrow{DM} = \overrightarrow{DA} + \overrightarrow{AM} = \overrightarrow{DA} + 3\overrightarrow{AB}$ | 2p |
| | $\overrightarrow{DN} = \overrightarrow{DA} + \overrightarrow{AN} = \overrightarrow{DA} + \frac{3}{4}\overrightarrow{AC}$ | 2p |
| | $\overrightarrow{DA} + \frac{3}{4}(\overrightarrow{AD} + \overrightarrow{AB}) = \overrightarrow{DA} - \frac{3}{4}\overrightarrow{DA} + \frac{3}{4}\overrightarrow{AB}$ | 2p |
| | $\overrightarrow{DA} - \frac{3}{4}\overrightarrow{DA} + \frac{3}{4}\overrightarrow{AB} = \frac{1}{4}\overrightarrow{DA} + \frac{3}{4}\overrightarrow{AB} = \frac{1}{4}(\overrightarrow{DA} + 3\overrightarrow{AB}) = \frac{1}{4}\overrightarrow{DM} \Rightarrow D, N, M \text{ coliniare}$ | 1p |